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Magnetocrystalline anisotropy of rocks and massive ores: a mathematical model study and its fabric implications

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Abstract—A simple mathematical model has been used to evaluate the influence of grain magnetocrystalline anisotropy and the scatter of crystallographic axes of grains on the magnetic anisotropy of rocks and massive ores whose carrier of magnetism is a magnetically uniaxial mineral of the type of pyrrhotite or hematite. The variation in magnetic anisotropy of rocks and ores whose carrier of magnetism displays the magnetocrystalline anisotropy greater than 100 is due to the variation in the preferred orientations of crystallographic axes, while the influence of the variation in the grain anisotropy on the rock (ore) anisotropy can be neglected.

INTRODUCTION

THE MOST important factor controlling the magnetic anisotropy of rocks is the preferred orientation either by grain shape or by crystal lattice of ferromagnetic minerals. Grain shape operates in rocks whose ferromagnetic fraction is represented by cubic titanomagnetite, while crystallographic orientation operates in rocks whose carrier of magnetism is a non-cubic mineral, e.g. hematite or pyrrhotite. While many aspects of the magnetic anisotropy of magnetite-bearing rocks have been treated, only minor attention has been paid to the magnetic anisotropy of rocks containing hematite or pyrrhotite.

The investigation of monocrystals has shown that the magnetocrystalline anisotropy of hematite and pyrrhotite is very high, with the degree of anisotropy (commonly defined as the ratio between the maximum and minimum susceptibilities) exceeding 100 (Uyeda *et al.* 1963, Schwarz 1974), whereas in the majority of hematite-bearing rocks it is less than 1.2 (Hrouda & Janák 1971, van den Ende 1977) and in pyrrhotite and hemo-ilmenite ores less than 3 (Hargraves 1959, Schwarz 1974). Consequently, the axes of hard and easy magnetization of hematite (pyrrhotite) grains must be very widely scattered in space. The present paper aims to evaluate this scatter quantitatively, using a simple mathematical model, and to compare the model scatter with the literary data on the c axis fabric obtained by means of the X-ray universal stage. In addition, the potential of magnetic anisotropy for the determination of the preferred orientation of crystals in hematite and pyrrhotite ores is briefly discussed.

A MODEL

Let us consider a model rock whose ferromagnetic fraction is represented by one type of mineral which possesses hexagonal or trigonal symmetry in its crystal lattice and exhibits high and isotropic susceptibility in its basal plane, and low susceptibility along the crystallographic c axis. In addition, it is assumed that the grains of this mineral are distributed homogeneously within a rock matrix of zero susceptibility; the crystals are spherical in shape, of equal size and so numerous that the distribution of their c axes can be described by a continuous function. Finally, the c axes of the grains are assumed to exhibit a preferred orientation with one maximum, possessing axial symmetry.

Let us introduce two rectangular coordinate systems for our model calculations. The first system is called the grain coordinate system and its axes x_1 and x_2 lie within the basal plane of a ferromagnetic grain, while the axis x_3 coincides with the crystallographic c axis. The second system is termed the specimen coordinate system and its axes y_1 and y_2 lie within the plane perpendicular to the axis of axial symmetry of the c-axis fabric, while the y_3 axis coincides with the axis of axial symmetry. These systems are related by a pair of conventional polar angles φ and ψ.

Let us assume each ferromagnetic grain to be placed in such a volume fraction of the matrix that the mean susceptibility of this volume element equals that of the whole rock specimen. The susceptibility tensor of the volume element, **K**, has the following shape in a general coordinate system:

$$\mathbf{K} = \begin{vmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{vmatrix}. \quad (1)$$

In the grain coordinate system, which coincides with the coordinate system of principal susceptibilities, the non-diagonal components are zero, while the diagonal components are in fact principal susceptibilities. In our case of a uniaxial grain, $K_{11} = K_{22} = K_1$ and $K_{33} = K_3$, $(K_1 > K_3)$. In the specimen coordinate system the susceptibility tensor of the volume element has the following components:

$$K_{11} = K_1(\cos^2\varphi + \sin^2\varphi \cos^2\psi) + K_3 \sin^2\varphi \sin^2\psi$$

$$K_{22} = K_1(\sin^2\varphi + \cos^2\varphi \cos^2\psi) + K_3 \cos^2\varphi \sin^2\psi$$

$$K_{33} = K_1 \sin^2\psi + K_3 \cos^2\psi$$
$$K_{12} = K_{21} = \sin\varphi \cos\varphi \sin^2\psi\ (K_1 - K_3) \quad (2)$$
$$K_{23} = K_{32} = \cos\varphi \cos\psi \sin\psi\ (K_1 - K_3)$$
$$K_{13} = K_{31} = \sin\varphi \sin\psi \cos\psi\ (K_3 - K_1).$$

If $f(\varphi, \psi)$ is a frequency density function characterizing the distribution of c axes of ferromagnetic grains on a surface of a unit hemisphere, then the susceptibility tensor of a specimen may be expressed in the specimen coordinate system as follows (see Owens 1974):

$$\mathbf{k} = \int_\varphi \int_\psi f(\varphi, \psi)\, \mathbf{K}(\varphi, \psi) \sin\psi \, d\varphi \, d\psi. \quad (3)$$

This expression is based on the assumption that either ferromagnetic grains are sufficiently far apart not to interact magnetically or, if they interact, that this interaction results only in a lowering of the mean susceptibility, without the shape of the susceptibility tensor being affected.

For the function $f(\varphi, \psi)$ a truncated Fisher (1953) distribution cut at $\psi = 90°$ can be adopted, since the distribution of the c axes is sufficiently described when the frequency density function for a hemisphere is known. Then, the Fisher frequency density function is modified to:

$$f = \kappa\, e^{\kappa \cos\psi}/2\pi(e^\kappa - 1), \quad (4)$$

where κ is measure of concentration of c axes around the mean direction.

After inserting the f value of Equation (4) in equation (3) and integrating, the following components of the susceptibility tensor of a specimen are obtained

$$k_{11} = k_{22} = (K_1 + K_3)/2 + I_2(K_1 - K_3)/2I_1$$
$$k_{33} = K_1 - I_2(K_1 - K_3)/I_1,$$
$$k_{12} = k_{21} = k_{23} = k_{32} = k_{13} = k_{31} = 0, \quad (5)$$

where

$$I_1 = (e^\kappa - 1)/\kappa$$

and

$$I_2 = (1/\kappa - 2/\kappa^2 + 2/\kappa^3)e^\kappa - 2/\kappa^3.$$

Introducing the mean (invariant) susceptibility, $\bar{K} = (2K_1 + K_3)/3$, equations (5) may be written

$$k_{11} = k_{22} = \bar{K} + (K_1 - K_3)(I_2/I_1 - 1/3)/2$$
$$k_{33} = \bar{K} - (K_1 - K_3)(I_2/I_1 - 1/3). \quad (6)$$

Equations 6 can be seen as a specific example of the general equation for uniaxial grains given by Owens (1974, equation 10):

$$k_{jj} = \bar{K} + (K_1 - K_3)I_{jj}. \quad (7)$$

In the case of high grain anisotropy, if $K_1 \gg K_3$, then

$$\bar{K} \doteq 2K_1/3, \quad K_1 - K_3 \doteq K_1$$

and

$$k_{jj} \doteq K_1(2/3 + I_{jj}). \quad (8)$$

Consequently, in the limit, any measure of magnetic anisotropy which is normalized (e.g. the degree of anisotropy, $P = k_1/k_3$) depends only on the orientation function and not on the grain anisotropy.

In the next Section, using the model given by equations (5) and (6), the accuracy of the limiting approximation with increasing grain anistropy will be investigated numerically.

RESULTS OF THE CALCULATION AND THEIR IMPLICATIONS

The results of the calculation are summarized in Fig. 1, where the degree of magnetic anisotropy of a specimen ($P_s = k_1/k_3$) is plotted as a function of the degree of magnetic anisotropy of crystals ($P_c = K_1/K_3$), and of the parameter κ. The ranges of P_c and κ have been chosen in such a way that the P_c and P_s values calculated correspond to those observed in natural minerals, rocks and massive ores (after Uyeda *et al.* 1963, Schwarz 1974, Janák 1973, Hargraves 1959).

It can be seen from Fig. 1 that the maximum values of the degree of anisotropy of a specimen increase with rising κ. Further, for $P_c < 10$, the P_s values increase rapidly with increasing P_c for all values of κ used. Within the range of $10 < P_c < 100$, the P_s values increase with increasing P_c as well, but for low κ the increase is very slow and for higher κ it is still comparatively rapid. For $P_c > 100$ and $\kappa < 4$ the degree of anisotropy of the specimen either increases very slowly or remains constant. The change of P_s is comparable with its determination error and lies within the range of between-specimen scatter of the majority of rocks and ores (for the scatters see Schwarz 1974, fig. 3 and Hargraves 1959, table 1). For $P_c > 100$ and $\kappa > 4$, the change of the P_s is clearly higher than its determination error, but it still lies within the scatter range.

Little investigation has been done on the magnetic anisotropy of monocrystals of magnetically uniaxial minerals in the low-field—for example, pyrrhotite and hematite. Uyeda *et al.* (1963) measured the low-field susceptibility anisotropy of monocrystals of ferrimagnetic pyrrhotite, and found the susceptibility to be high and isotropic within the basal plane, and very low along the ternary axis, with the $P_c > 100$. Schwarz (1974) found the susceptibility of ferrimagnetic Fe_7S_8 pyrrhotite to be about 1.3 (in the SI system of units) in the basal plane and about 6.3×10^{-4} along the c axis; thus the P_c is about 2000. The low-field susceptibility anisotropy of hematite was studied by Uyeda *et al.* (1963), while other authors (e.g. Porath & Raleigh 1967) investigated the high-field anistropy. Uyeda *et al.* (1963) found the susceptibility in the basal plane to be virtually isotropic, ranging from 10^{-2} to 10^{-1}, while along the c axis it was hardly detectable. The P_c parameter exceeded 10 in all cases, and in the majority of crystals it was higher than 100.

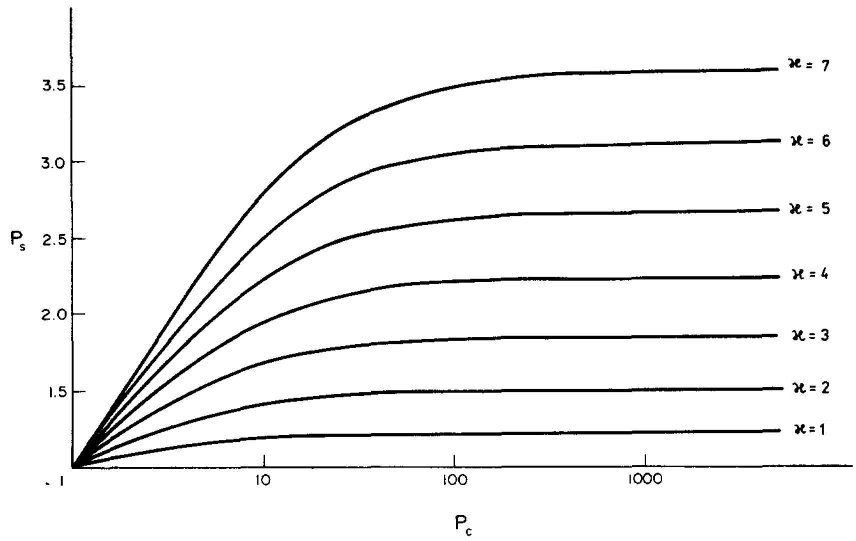

Fig. 1. Diagram to show the dependence of the degree of magnetic anisotropy of a model rock (P_s) on the degree of anisotropy of a ferromagnetic mineral (P_c) carrying the magnetism of the rock and on the parameter κ characterizing the intensity of concentration of the c axes along their mean direction.

Our model calculation has shown that in rocks for whose carrier of magnetism $P_c > 100$, the degree of anisotropy is controlled almost exclusively by the scatter of the c axes of the ferromagnetic grains, while the effect of the degree of magnetocrystalline anisotropy of the carrier of magnetism is very small and may be neglected in practice. Since in natural hematite and pyrrhotite, as shown above, $P_c > 100$, there is no need to investigate precisely the P_c for a particular hematite (pyrrhotite) in each rock studied, in order to interpret the magnetic anisotropy of rocks with hematite or pyrrhotite in quantitative terms of preferred orientation of these minerals. This conclusion is important, because it is very difficult, and in fact practically impossible, to measure the low-field magnetic anisotropy of the carriers of magnetism in rocks, which are usually very small crystals.

Table 1 shows the degrees of magnetic anisotropy of some hematite and pyrrhotite bearing rocks as well as hemo-ilmenite and pyrrhotite ores. It is clear from Table 1 that the degree of anisotropy of the hematite-bearing rocks is very low, which can be accounted for either by a very slight preferred orientation of the c axes of the hematite crystals or by the so-called 'masking' effect of a nearly isotropic paramagnetic fraction, which lowers the resultant anisotropy (see Fuller 1963). In the former case, the corresponding κ value is less than 0.5, which is very small ($\kappa = 0$ indicates a uniform distribution of the c axes). Consequently, the mechanism orientating the basal planes of hematite in red beds is only very weak. The degree of anisotropy of strongly deformed slates is higher (Rathore 1979), thus indicating that deformation is a more effective orientation process.

In ores the degree of anisotropy is considerably higher; this indicates a relatively inten ive preferred orientation of the basal planes of ore grains. In the ores cited in Table 1 the k_1/k_2 ratio is much smaller than the k_2/k_3 ratio, which indicates that the c axis fabric exhibits one maximum and possesses almost axial symmetry; it is similar to that used in our model calculations.

An attempt has been made to calculate the theoretical magnetic anisotropy of pyrrhotite ore for which the distribution of the c axes is known, on the assumption that this distribution is similar to that described by equation (4). For this purpose, recourse was had to the results of the X-ray textural investigations made by Bayer & Siemes (1971) of the pyrrhotite ore of the Bayerland (FRG), Röros (Norway) and Kambalda (Australia) deposits. The pole figures of the (10$\bar{1}$0) planes (Bayer & Siemes 1971, figs. 4c, 5c and 6c) were transformed into the pole figures of (0001) and simplified in such a way that the c axis fabric possessed axial symmetry. From these simplified pole figures empirical distributions of the c axes were constructed as well as the theoretical curves for some κ values; both are presented in Fig. 2. The theoretical curves were constructed using the formula

$$F = (e^{\kappa} - e^{\kappa \cos \psi})/(e^{\kappa} - 1), \quad (9)$$

which follows from the integration of equation (4). An

Table 1. The degree of magnetic anisotropy in some hematite-bearing rocks and massive ores

Rock	Region	P_s	Totals of specimens	Author
Red beds	Moravian Karst	1.034	24	Hrouda & Janák (1971)
Red beds	Dome de Barrot	1.050	145	van den Ende (1977)
Slate with hematite	N. Wales	1.182	275	Rathore (1979)
Slate with pyrrhotite	Wales	1.367	65	Fuller (1963)
Hemo-ilmenite ore	Allard Lake	2.176	51	Hargraves (1959)
Pyrrhotite ore	Sudbury area	1.671	120	Schwarz (1974)

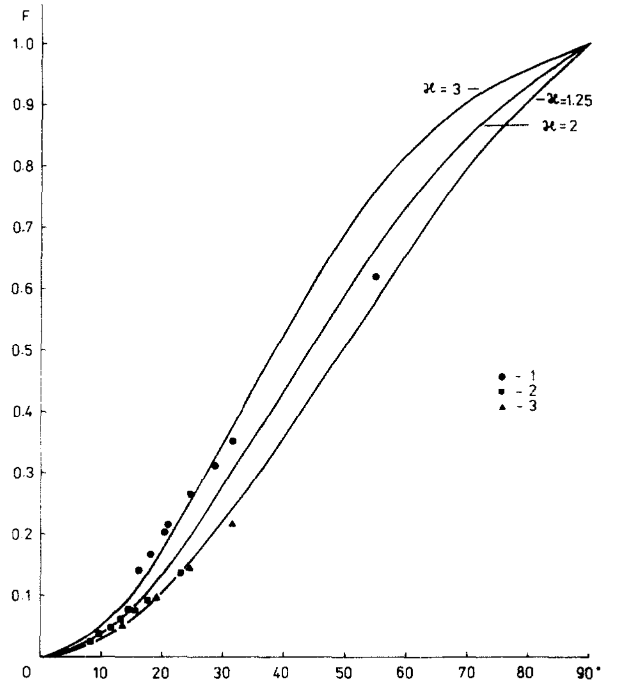

Fig. 2. Diagram to show the empirical distributions of the c axes of pyrrhotite in some ores (after Bayer & Siemes 1971) and the corresponding theoretical distributions according to formula (g). Legend: 1—Bayerland Deposit (FRG), 2—Röros Deposit (Norway), 3—Kambalda Deposit (Australia).

attempt was made to obtain a reasonably good fit of the empirical and theoretical distributions by matching κ. It is clear from Fig. 2 that the c axis distribution described by equation (4), even though it does not fit the natural distribution perfectly, can in first approximation be regarded as representing a c axis distribution of pyrrhotite in the massive ore. For the Bayerland deposit $\kappa = 3$, which yields $P_s = 1.85$, for the Röros deposit $\kappa = 1.25$ ($P_s = 1.3$) and for the Kambalda deposit $\kappa = 2$ ($P_s = 1.5$). From a comparison of the calculated values of the degree of anisotropy with the degrees of anisotropy of the ores measured (Table 1), it can be deduced that our model seems to be a realistic approximation of the natural situation. Although it does not consider symmetry lower than axial, it can serve as the basis for a rough quantitative comparison of the petromagnetic and X-ray textural studies of ores. Further, the calculations have shown that magnetic anisotropy can be employed for investigating the c axis fabric of massive hematite and pyrrhotite ores.

CONCLUSIONS

Using a simple mathematical model, simulation of the magnetocrystalline anisotropy of hematite or pyrrhotite-bearing rocks and ores has been made for an anisotropy range corresponding to that of natural rocks and ores. The modelling has enabled the following conclusions to be drawn:

1. If the degree of anisotropy of mineral carrying the magnetism of a rock exceeds 100, the degree of anisotropy of a rock is controlled almost exclusively by the scatter of the c axes and the precise mineral magnetocrystalline anisotropy need not be known for the interpretation of rock anisotropy.
2. The preferred orientation of the basal planes of hematite in red beds is developed only slightly, thus testifying to an ineffective mechanism orientating the hematite basal planes in these rocks. The preferred orientation of the basal planes of hemo-ilmenite and pyrrhotite in massive ores is considerably higher, and is easy to detect by magnetic anisotropy measurements.
3. The theoretical magnetic anisotropy of some pyrrhotite ores with a known distribution of c axes is comparable to that measured on pyrrhotite ore. Hence magnetic anisotropy appears to be a potential tool for c axis fabric analysis in massive pyrrhotite and hematite ores.

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REFERENCES

Bayer, H. & Siemes, H. 1971. Zur Interpretation von Pyrrhotin-Gefügen. *Miner. Deposita* **6**, 225–244.
Ende, C. van den 1977. Palaeomagnetism of Permian red beds of the Dome de Barrot (S. France). Ph.D. thesis, University of Utrecht.
Fisher, R. 1953. Dispersion on a sphere. *Proc. R. Soc.* **A217**, 295–305.
Fuller, M. D. 1963. Magnetic anisotropy and paleomagnetism. *J. geophys. Res.* **68**, 293–309,
Hargraves, R. B. 1959. Magnetic anisotropy and remanent magnetism in hemo-ilmenite from ore deposits at Allard Lake, Quebec. *J. geophys. Res.* **64**, 1565–1578.
Hrouda, F. & Janák, F. 1971. A study of the hematite fabric of some red sediments on the basis of their magnetic susceptibility anisotropy. *Sediment. Geol.* **6**, 187–199.
Janák, F. 1973. A brief outline of the magnetic susceptibility anisotropy of various rock types. *Studia geophys. geod.* **17**, 123–130.
Owens, W. H. 1974. Mathematical model studies on factors affecting the magnetic anisotropy of deformed rocks. *Tectonophysics* **24**, 115–131.
Porath, H. & Raleigh, C. B. 1967. An origin of the triaxial basal-plane anisotropy in hematite crystals. *J. appl. Phys.* **38**, 2401–2402.
Rathmore, J. S. 1979. Magnetic susceptibility anisotropy in the Cambrian Slate Belt of North Wales and correlation with strain. *Tectonophysics* **53**, 83–97.
Schwarz, E. J. 1974. Magnetic fabric in massive sulphide deposits. *Can. J. Earth. Sci.* **11**, 1669–1675.
Uyeda, S., Fuller, M. D., Belshé, J. C. & Girdler, R. W. 1963. Anisotropy of magnetic susceptibility of rocks and minerals. *J. geophys. Res.* **68**, 279–292.